CRCM internal variability and the singular vector technique

A case study: July 1993

Emilia Diaconescu, René Laprise and Ayrton Zadra

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Outline:

- CRCM Internal Variability: 17 - 20 July 1993
- Singular Vector (SV) technique
- Projection on SV base
- Conclusions and future work
**20 simulations (Alexandru et al., 2007)**

- **Model:** CRCM v. 3.6.1
- **18 vertical levels**
- **120x120 grid points; Δx=45 Km**
- **Physics:** GCMii
- **Time step:** 15 min
- **Pilot:** NCEP re-analyses
- **IC:** 01 to 20 May 1993
- **Analyses period:** July 1993

**Internal Variability:**  
\[ IV(X) = \iint_{vol} \frac{1}{M-1} \sum_{m} X'_m^2 \]

\[ X'_m = X_m - \frac{1}{M} \sum_{m} (X_m) \]

\[ X'_m = \{U'_m, V'_m, T'_m, P'_s\} \]
**CRCM Internal Variability:**

20 July 00Z
Sea level pressure

**20 simulations (Alexandru et al., 2007):**
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**Internal Variability:**

\[ IV(X) = \frac{1}{M} \sum_{m} \frac{1}{M-1} \sum_{m} X'_m^2 \]

\[ EM = \frac{1}{M} \sum_{m} (X_m) \]

\[ X'_m = X_m - \frac{1}{M} \sum_{m} (X_m) \]

\[ \{U'_m, V'_m, T'_m, ps'_m\} \]

**Perturbations**

**Total energy of perturbation “m”:**

\[ Et_m = \iint_{Vol} \rho (U'_m^2 + V'_m^2) dV + \iint_{Vol} \rho \left( \frac{C_p}{T^r_m} T'_m^2 \right) dV + \iint_{surface} \left( \frac{RT_r}{p_r g} ps'_m^2 \right) dA \]

**Kinetic term**

**Potential term**

**Surface-pressure term**
**CRCM Internal Variability:**

20 simulations (Alexandru et al., 2007)

- **Model:** CRCM v. 3.6.1
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**Internal Variability:**

\[ IV(X) = \frac{1}{M} \frac{1}{\sum_{m} X_m} \]  

\[ EM = \frac{1}{M} \sum_m (X_m) \]

\[ X'_m = X_m - \frac{1}{M} \sum_m (X_m) \]

\[ X'_m = \{U'_m, V'_m, T'_m, ps'_m\} \]

Average total energy of perturbations:

\[ Et = \frac{1}{M} \sum_m (Et_m) = \frac{1}{M} \sum_m \left[ \int_{Vol} \rho(U'_m^2 + V'_m^2) dV + \int_{Vol} \rho \frac{C_p}{T} T'_m^2 dV + \int_{surface} \left( \frac{RT}{p_s g} ps'_m^2 \right) dA \right] \]  

- **Kinetic term**
- **Potential term**
- **Surface-pressure term**
CRCM Internal Variability:

- Kinetic energy
- Potential energy
- Surface-pressure energy

Energy vs. July
Objective: Understanding of processes leading to episodes of high IV.

Tools: Singular Vectors = a set of perturbations with the most rapid linear growth with respect to a given norm, over an interval of time.

Can IV be expressed in terms of rapid linear-growing perturbations?
CRCM Internal Variability:

Objective: Understanding of processes leading to episodes of high IV.

Tools: Singular Vectors = a set of perturbations with the most rapid linear growth with respect to a given norm, over an interval of time.

Can IV be expressed in terms of rapid linear-growing perturbations?
Singular Vectors (SVs): 

$$\mathbf{x}_0 = \{u, v, T, \rho\} \quad \mathbf{x}(t) = M(t, 0)\mathbf{x}_0 \quad M(t, 0) = \text{tangent-linear model}$$

$$\|\mathbf{x}\|^2 = (\mathbf{x}, \mathbf{x}) = \langle \mathbf{x}, E\mathbf{x} \rangle = \iiint_V \rho(u^2 + v^2) dV + \iiint_V \rho \left(\frac{\mathbf{c}_p}{T} T^2\right) dV + \iint_A (\frac{R \mathbf{T}_r}{\mathbf{p}_r \mathbf{g}}) \rho^2 dA$$
Singular Vectors (SVs):

\[ x_0 = \{ u, v, T, ps \} \quad x(t) = M(t,0)x_0 \quad M(t,0) = \text{tangent-linear model} \]

\[
\|x\|^2 = (x,x) = \langle x, Ex \rangle = \iiint \rho(u^2 + v^2) dV + \iiint \rho \left( \frac{c_p}{T_r} T^2 \right) dV + \int_A \left( \frac{RT_r}{p_r g} ps \right)^2 dA
\]

Amplification factor \quad Optimization time

\[
\sigma^2 = \frac{\|x(t)\|^2}{\|x_0\|^2} = \frac{\langle x(t), E_t x(t) \rangle}{\langle x_0, E_0 x_0 \rangle} = \frac{\langle Mx_0, E_t Mx_0 \rangle}{\langle x_0, E_0 x_0 \rangle} = \frac{\langle x_0, M^T E_t Mx_0 \rangle}{\langle x_0, E_0 x_0 \rangle}
\]

Tangent linear adjoint

\[
M^T E_t M \quad v_i = \sigma_i^2 E_0 v_i
\]

\[ v_i = \text{singular vectors} \quad \sigma_i = \text{singular values} \]
### Singular Vectors:

**Choices to be made in the SV calculation:**
- initial conditions
- optimization time interval \((t-t_0)\)
- tangent linear and adjoint operators
- initial and final time norm \(E_0\) and \(E_t\)
- initial and final domains
Singular Vectors:

Choices to be made in the SV calculation:

- **initial conditions** = CRCM ensemble mean for 17 July 00Z
- **optimization time interval (t\(-t_0\))** = 24 h
- **tangent linear and adjoint operators** = GEM (400 x 200 uniform Gaussian grid and 58 eta levels)
- **initial and final time norm** $E_0$ and $E_t$ = dry total energy norm
- **initial and final domains** = sub-domains of CRCM
Energy partition:

- **Kinetic term**
- **Potential term**
- **Surface-pressure term**

Initial energy partition, with total energy normalized as one.

Final energy partition, with all components normalized by total initial energy.

Amplification factor > 1.

Singular vectors: SV1, SV2, SV3, SV4, SV5, SV6, SV7, SV8, SV9, SV10.
Energy partition:

- **Kinetic term**
- **Potential term**
- **Surface-pressure term**

**Initial energy partition**, with total energy normalized as one.

**Final energy partition**, with all components normalized by total initial energy.
17 July; zonal wind (m/s)

17 July; meridional wind (m/s)

File: ppm_moy_ens_Select Field: U Time: 1536 Level: 850

File: ppm_moy_ens_Select Field: V Time: 1536 Level: 850

CRCM ensemble mean

Perturbation “n”

Leading SV
17 July; zonal wind (m/s)

17 July; meridional wind (m/s)

Perturbation "n" CRCM ensemble mean Leading SV
Perturbation "n" CRCM ensemble mean
17 July; zonal wind (m/s)

17 July; temperature (°C)
Perturbation "n" Leading SV
Projection of CRCM perturbations ($X_k$) on the orthonormal base of initial SVs $\{\hat{Y}_j, j=1, \ldots, 10\}$.

$$\alpha_{X_kY_j} = \left\langle X_k, E\hat{Y}_j \right\rangle$$

Projection matrix

$$M = \left[ m_{k,j} ; m_{i,j} = \frac{\alpha_{k,j}^2}{\|X_k\|^2} \right]$$
Projection of CRCM perturbations ($X_k$) on the orthonormal base of initial SVs ${\hat{Y}_j, j=1, \ldots 10}$.

$$\alpha_{X_k,Y_j} = \left<X_k, E\hat{Y}_j\right>$$

Projection matrix

$$M = \left[ m_{k,j} \right]; m_{i,j} = \frac{\alpha_{k,j}^2}{\|X_k\|^2}$$

Percentage of CRCM-perturbation initial total energy explained by SVs

- On average: 21%
Conclusions:

CRCM Perturbations:

- Barotropic structure.
- Total energy dominated by kinetic term.

SVs:

- At initial time, potential energy is greater than kinetic energy, while at final time kinetic energy prevails.

- An average of 21 % of the CRCM-perturbation total energy can be explained by initial SVs.

- The projection is primarily made on the first two SVs characterized by an amplification factor greater than 1.
Future work:

Analyze to which extent SVs can explain the IV developed in winter conditions, when baroclinic systems are preponderant.

- Model: GEM-LAM
- N. American domain
- 10 simulations
- IC: 01 to 10 November 1992
- Analyses period: December 1992

Sea surface pressure; 10 simulations
Fields from Léo Separovic
Questions?

Thank you for your attention
The dry tangent-linear model of GEM:

-Linearization of the model’s dynamical core (the set of primitive equations)
-Linearization of the physics:
  1) a representation of vertical diffusion which consists in a simplified planetary boundary layer parameterization
  -this prevents the development of non-meteorological instabilities near the surface
  2) the subgrid-scale orographic drag which describes gravity-wave drag and low-level blocking
  3) the stratiform precipitation

The moist tangent-linear model of GEM:

-the dry tangent-linear model of GEM
-convective precipitation

The linearization of physics is particularly difficult because parametrizations of physical processes often are highly nonlinear.

Total energy: horizontal distribution

17 July 1993

**CRCM ensemble**
Max: $2 \times 10^5$ J/m$^2$
**SV set**
Max: $13 \times 10^6$ J/m$^2$

18 July 1993

**CRCM ensemble**
Max: $4 \times 10^5$ J/m$^2$
**SV set**
Max: $3 \times 10^6$ J/m$^2$
Projection of ensemble-A perturbations ($X_k$) on the orthonormal base of SVs \{ $\hat{Y}_j$, j=1, ...,10\}.

\[
\hat{Y}_j = \frac{Y_j}{|Y_j|} = \frac{Y_j}{\sqrt{Y_j, EY_j}}
\]

\[
\alpha_{X_kY_j} = \left\langle X_k, E \hat{Y}_j \right\rangle
\]

\[
\tilde{X}_k = \sum_j (\alpha_{kj} \hat{Y}_j)
\]

\[
\|\tilde{X}_k\|^2 = \left\langle \tilde{X}_k, E\tilde{X}_k \right\rangle = \sum_j \alpha_{kj}^2 \left\langle \hat{Y}_j, E\hat{Y}_j \right\rangle = \sum_j \alpha_{kj}^2
\]

Projection matrix

The percentage of the total energy of each k CRCM perturbation explained by the set of SVs:

\[
\|\tilde{X}_k\|^2 = \sum_j \alpha_{kj}^2
\]

\[
\|X_k\|^2 = \frac{\sum_j \alpha_{kj}^2}{\left\langle X_k, EX_k \right\rangle}
\]